

**Fig. 1 Reattachment pressure coefficient.**

Figure 1 shows values of the reattachment pressure coefficient for three values of  $X/\sigma\delta^{**}$ . ( $X$  is the length of the constant pressure mixing region before reattachment,  $\sigma$  is the customary similarity parameter, and  $\delta^{**}$  is the boundary-layer momentum thickness at separation.) The highest values of  $C_{PR}$  correspond to reattachment of fully developed free shear layers which are independent of initial boundary-layer effects (similar solutions). Intermediate values of  $C_{PR}$  are for the developing (or preasymptotic) shear layers. The theoretical curves shown in Fig. 1 are for free isoenergetic reattachments in air ( $k = 1.4$ ) calculated by available methods.<sup>2</sup>

Batham's recent correlation is plotted in Fig. 1. It is apparent that his correlation is for experimental cases in which the shear layer is not yet fully developed. Also one notes that Batham's correlation is for the reattachment point pressure only. Once the reattachment point pressure is known, this must be related to other flow conditions in order to determine a unique solution. An example of such a complete reattachment criterion has been given.<sup>2</sup>

#### References

- <sup>1</sup> Batham, J. P., "A Reattachment Criterion for Turbulent Supersonic Separated Flows," *AIAA Journal*, Vol. 7, No. 1, Jan. 1969, pp. 154-156.
- <sup>2</sup> Page, R. H., Kessler, T. J., and Hill, W. G., "Reattachment of Two-Dimensional Supersonic Turbulent Flows," Paper 67-FE-20, 1967, American Society of Mechanical Engineers.

## Reply by Author to R. H. Page

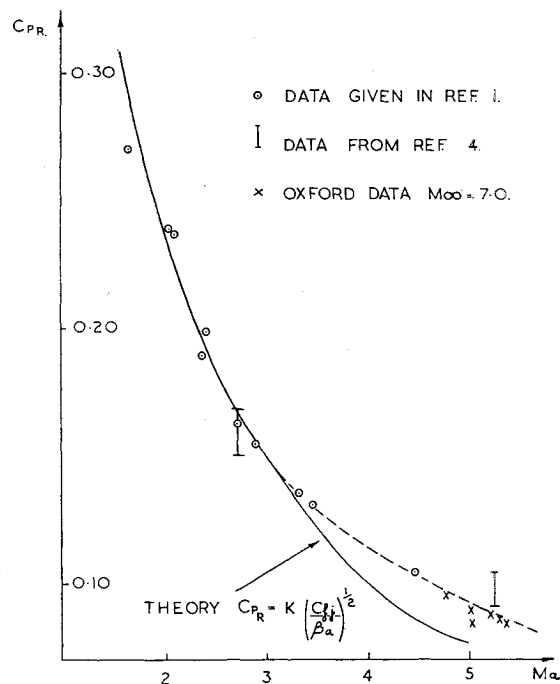
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**P**ROFESSOR Page presents a correlation of reattachment pressure coefficient in terms of the similarity parameter for similar and nonsimilar flows. However, this parameter is only applicable to flows where the constant pressure shear layer can be represented by a similar (or asymptotic) velocity profile.

The criterion proposed<sup>1</sup> is applicable for large values of  $X/\delta^{**}$ , where an asymptotic error function velocity profile can be used for the shear layer upstream of reattachment. The experiments of Sirieix et al.<sup>2</sup> show that under these conditions a similar reattachment pressure distribution is obtained.

It is apparent that a large discrepancy exists between the proposed criterion and the correlation of Page et al.<sup>3</sup> based on experiments on rearward facing steps. The correlation of Page et al. gives a relation between the velocity ratio on the discriminating streamline and the angle turned through at reattachment as a fraction of the total turned through at



**Fig. 1 Reattachment pressure coefficient.**

recompression. This discrepancy could be due to the use of asymptotic velocity profiles in calculating the velocity ratio on the discriminating streamline for small step heights where the shear layer may not have been similar, and the use of the turning angle downstream of reattachment in the correlation. The experiments of Sirieix et al.<sup>2</sup> and Roshko and Thomke<sup>4</sup> show that conditions downstream of reattachment may be varied independently of the pressure distribution up to reattachment.

The proposed criterion appears to give a good correlation of data obtained in a large number of facilities. Included in Fig. 1 are two points taken from the data of Roshko and Thomke,<sup>4</sup> where the reattachment point could easily be determined, and some recently obtained data for compression corner flows obtained in the Oxford University Hypersonic Gun Tunnel where the reattachment point was located by means of a surface Pitot. It would not appear to be possible at present to correlate flows with small values of  $X/\delta^{**}$  owing to the difficulty in evaluating the shear-layer velocity profile.

A solution to the over-all flowfield can be obtained by combining the reattachment criterion with the methods proposed by Childs et al.<sup>5</sup> or Roberts<sup>6</sup> as shown in Ref. 7.

#### References

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- <sup>2</sup> Sirieix, M., Mirande, J., and Delery, J., "Experiences Fondamentales sur le Recollement Turbulent d'un Jet Supersonique," *AGARD Conference Proceedings No. 4*, 1966, pp. 353-391.
- <sup>3</sup> Page, R. H., Kessler, T. J., and Hill, W. G., "Reattachment of Two-dimensional Supersonic Turbulent Flows," American Society of Mechanical Engineers Paper 67-FE-20, Chicago, 1967.
- <sup>4</sup> Roshko, A. and Thomke, G. J., "Observations of Turbulent Reattachment behind an Axisymmetric Downstream-Facing Step in Supersonic Flow," *AIAA Journal*, Vol. 4, No. 6, June 1966, pp. 975-980.
- <sup>5</sup> Childs, M. E., Paynter, G. C., and Redeker, E., "The Prediction of Separation and Reattachment Flow Characteristics for Two-dimensional Supersonic and Hypersonic Turbulent Boundary Layers," *AGARD Conference Proceedings No. 4*, 1966, pp. 325-352.

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<sup>6</sup> Roberts, J. B., "On the Prediction of Base Pressure in Two-Dimensional Supersonic Turbulent Flow," R and M 3434, 1966, Aeronautical Research Council.

<sup>7</sup> Batham, J. P., "Analysis of Turbulent Supersonic Separated Flows," 30, 336, FM 3964, May 1968, Aeronautical Research Council.

## Comment on "Numerical Lifting-Surface Theory—Problems and Progress"

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IN a recent paper, Landahl and Stark<sup>1</sup> present a progress report on the status of numerical approaches to non-steady lifting-surface theory for planar and nonplanar configurations as applied to the linearized thin-wing problem, with particular stress on the subsonic case.

Over the past few years, in the course of studies adapting unsteady lifting-surface theory to marine propellers,<sup>2-4</sup> Davidson Laboratory has developed a new method for the solution of the downwash surface integral equation. By proper expansion of the kernel function and introduction of the so-called "generalized lift operator," the chordwise integration is performed analytically with the additional advantage that the numerical solution is greatly simplified. These studies indicate that use of the generalized lift operator, which is in fact dictated by the nature of the integral equation itself, is a more accurate and rapid procedure than the "usual" numerical approaches for evaluating the steady and unsteady pressure distributions on lifting surfaces and resultant hydrodynamic forces.

This technique has been used in Ref. 5, where the lifting surfaces are the blades of a marine propeller operating in non-uniform inflow, and in Ref. 6 for the case of a deeply submerged, flat, rectangular hydrofoil in steady flow. In Ref. 7, this new approach has been applied to several two-dimensional, unsteady airfoil problems and has yielded results identical to the known explicit solutions.

### References

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<sup>2</sup> Shioiri, J. and Tsakonas, S., "Three-Dimensional Approach to the Gust Problem for a Screw Propeller," *Journal of Ship Research*, Vol. 7, No. 4, April 1964, pp. 29-53.

<sup>3</sup> Tsakonas, S. and Jacobs, W. R., "Unsteady Lifting Surface Theory for a Marine Propeller of Low Pitch Angle with Chordwise Loading Distribution," *Journal of Ship Research*, Vol. 9, No. 2, Sept. 1965, pp. 79-101.

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<sup>6</sup> Henry, C. J., "Application of Lifting Surface Theory to the Prediction of Hydroelastic Response of Hydrofoil Boats," Rept. 1202, Feb. 1967, Davidson Lab., Stevens Institute of Technology.

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## Reply by Authors to S. Tsakonas

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WE are grateful to S. Tsakonas for having drawn our attention to the interesting developments in lifting-surface theory at Davidson Laboratory. The successful application of the generalized lift operator, a simplified version of the variational method,<sup>1</sup> seems very valuable. Such an analytic chordwise integration is of course desirable also in the case of oscillating aircraft wings. Therefore, it should be valuable to know whether a similar quadrature process could be developed for compressible flow as well. There is one apparently unsettled question, however, namely that of the poor convergence of the lift distributions shown in Ref. 2.

### References

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## Comment on "Transonic Flow in Unconventional Nozzles"

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AN effort to derive the equations published in an analysis by Hopkins and Hill<sup>1</sup> pertaining to the transonic flow regime in annular and other unconventional nozzles has yielded some discrepancies. We believe the equations are correct as written in the following. The numbering and the nomenclature of the equations correspond to the numbering and nomenclature of the equations in the original paper.

$$(H/H_R)^2 = 1 + (M_R^2 q_1 - q_1) \Delta \eta + [q_1^2 - q_2 + (M_R^2/2)(2q_2 - q_1^2) + (\gamma/2)M_R^4 q_1^2] \Delta \eta^2 + [2q_1 q_2 - q_1^3 + (M_R^2/2)(q_1^3 - 2q_1 q_2 + 2q_3) + \gamma M_R^4 q_1 q_2] \Delta \eta^3 \quad (18)$$

$$x = \xi - \frac{H_R^2 \eta_R^2}{2Y_R^3} (2H_R H_R' Y_R - H_R^2 \sin \omega) \Delta \eta^2 + \left[ \frac{H_R^4 \eta_R^3}{3Y_R^5} \cos \omega (4H_R H_R' Y_R - \frac{5}{2} H_R^2 \sin \omega) - \frac{H_R^3 H_R' \eta_R}{Y_R^2} + \frac{H_R^4 \eta_R \sin \omega}{2Y_R^3} \right] \Delta \eta^3 \quad (20)$$

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